Review of Stormwater Quality, Quantity and Treatment Methods
Part 1: Stormwater Quantity Modelling

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Abstract

A review of stormwater quantity and quality in the urban environment is presented. The review is presented in three parts. The first part reviews the mathematical methods for stormwater quantity and has been undertaken by examining a number of stormwater models that are in current use. The important feature of models, their applications, and management has been discussed. Different types of stormwater management models are presented in the literatures. Generally, all the models are simplified as conceptual or empirical depending on whether the model is based on physical laws or not. In both cases if any of the variables in the model are regarded as random variables having a probability distribution, then the model is stochastic model. Otherwise the model is deterministic (based on process descriptions). The analytical techniques are presented in this paper.

Keywords: Stormwater, Quantity and quality, Mathematical models, Analytical technique

1. Introduction

It is estimated that by year 2025, half of the world’s population will live in urban areas. As the land occupations by urban areas are very small compared to the rural areas, the human activities intensify local competition for all types of resources, with water amongst the most vital.

The development of water resources requires the conception, planning, construction and operation of facilities to control and utilize water for a variety of purposes. Stormwater infiltration is an example of use of urban excess water that will not cause excessive damage to property loss of life and inconvenience of people and the receiving aquatic bodies.1-4)

Water resource managers are faced not only to control and management of runoff water quantity but with the maintenance of water quality as well, especially during the wet weather period.5,6) Precipitation falling over an urban watershed passes through an enormously complex hydrologic and hydraulic system. As it moves through this system it concentrates into larger and larger flow streams and picks up a wide variety of pollutants in the process. An urbanized area is, by definition, an area of concentrated human activity. With this activity comes an increase in runoff volumes and flow rate due to covering of much of the surface with impervious materials such as concrete, asphalt etc.7) In addition, such a concentration of human activity can only be maintained by a large influx of a great variety of materials. Some of this waste is transported from the urban area by stormwater runoff to receiving waters.8,9) This transport process is very efficient since urban areas have elaborate drainage systems to remove runoff quickly. This is made complicated in prediction by unequal distribution of water and its availability at any place varying with time.10)

Engineers have viewed urban stormwater with different perspectives over the years. In earlier times, the concern was for flood control and removing runoff as expeditiously a possible.11,12) In more recent times, the cross purposes of removing runoff from streets and parking lots and yet not overwhelming receiving waters led to the notion of comprehensive stormwater management.13) Everyday increasing legislation parameters enforces scientists and engineers to predict and do maintenance of water quantity and quality using different approaches. Application of computer models of urban stormwater flow and quality have been extremely useful in establishing whether various management strategies produce water quality that conforms to the legislation. However, simulation of urban runoff quality is very dif-
ficult in getting accuracy. Previous researchers have discussed many difficulties of simulation of urban runoff quality. Very large uncertainties arise both in the representation for the physical, chemical and biological process and in the acquisition of data and parameters for the model. The real mechanism of pollutants build-up in dry weather period involves factors such as wind, traffic, atmospheric fall out, land surface activities erosion, street cleaning and other imponderables. Many efforts have been made to estimate the amount of pollutants on the surface at the beginning of rainfall event and the pollutants loaded during runoff by using some physically based equations. The uncertainties can be dealt by collecting sufficient data to calibrate the model equation for qualitative simulation. Hundreds of models have been developed and practiced in stormwater management. In this review paper several types of models frequently appear in the literatures are summarized. This review illustrates the diversity of approaches and parameters that are considered in urban stormwater models. The review will try to cover both quality and quantity models and also discusses some treatment practices.

2. Modelling Approaches

Different types of stormwater management models are developed in the literatures. Generally, all the models are simplified as conceptual or empirical depending on whether the model is based on physical laws or not. In both cases if any of the variables in the model are regarded as random variables having a probability distribution, then the model is stochastic model. Otherwise the model is deterministic (based on process descriptions). Commonly used stochastic techniques are regression, transfer function, neural networks and system identifications. The stochastic model may produce the different response each time due to selection of random variables. In case of deterministic model, the result is always identical for the same input parameters. These models try to represent the physical processes observed in real world. Most of the urban runoff models are deterministic models. Furthermore, the deterministic models can be a single event or continuous process driven. Event model are short term models used for simulating a few or individual storm events. Continuous models simulate a catchment’s over-

3. Urban Runoff Quantity Problems and Models

Rain falling over an urban watershed will strike either a pervious surface or an impervious surface. On pervious surface most of the rainfall infiltrated to the subsurface and some remains runoff as overland flow and depression storage. Depression storages are small pores on land surfaces which temporarily store water. Some portion of the runoff water may be evaporated. On impervious surface nearly all the rainfall becomes runoff due to lack of infiltration and very limited depression loss. Surface runoffs from both pervious and impervious surfaces find the way to channels streams. The runoff behaviour of stormwater varies according to the surface types. Urban drainage network channels try to overcome the runoff to much extent by accommodating the generated runoff within it. But the increased human activity and their produced conditions such as imperviousness and manmade water courses lead faster rainfall to runoff transformation resulting deleterious effects such as flooding, stream erosion, habitat destruction etc.

There are three different approaches to urban stormwater modelling: namely, the design storm event approach, the continuous simulation approach, and derived probability distribution approach. There are several models from simple to complex. Computer aided models have been used to simulate the behaviour of aquatic systems since the mid 1960’s. Good model simulator appeared in the earlier 1970’s and were developed primarily by US government agencies such as Environmental Protection Agency. Since then number of urban watershed models have been deve-
developed and introduced. These models include from simple conceptual models to very complex hydraulic model.

Simple models require less data; calculations are not repetitive and may require simple calculations. The output of the calculations may provide limited information on flow and the pollutants. In the case of complex models, the routing (flowing) behavior is based on some physical laws describing the flow within the catchments. Depending on the parameters introduced they describe the behavior of catchment in different complexities. Generally there are three different approaches to urban stormwater modelling; namely, the design storm event approach, the continuous simulation approach and derived probability distribution approach. The design storm event approach is simple and does not require the historical data. The model is based on single event data. Although it may simulate a single event and approximate rainfall runoff transformation of but it has many limitations. The model can not adopt the antecedent dry weather period. Moreover, if the rainfall is intermittent, the depression loss or storage loss can not be accommodated and model prediction fails. Long term performance is really critical for the model simulation. The continuous simulation approach involves conceptual modelling of the physical system, recognizing not only the properties of the storm but also the accumulative effect closely-spaced storms. One of the major drawbacks of continuous simulation modelling is its computational burden, resulting from a large number of simulation runs to calibrate and validate a representative number of system configurations.

Derived probability distribution approach is based on the probability density function of runoff characteristics in contrast to the utilization of the probability distribution function of rainfall characteristics.

The primary benefit of the analytical models lies in the availability of explicit mathematical solutions to performance measure. The analytical models are developed with derived probability distribution theory. Probability density functions (PDFs) describing the input meteorology to the system are transformed to PDFs of the system performance parameters by relationship describing the system hydrology and hydraulics.

4. Approaches to Stormwater Quantity Estimation

4.1. Simple Statistical and Empirical Models

Statistical models are normally developed with well-established multivariate pattern recognition techniques such as factor analysis, cluster analysis, correlation analysis etc. Other frequently used techniques for developing stormwater quality models include linear, non-linear and stepwise regression equations. These models relate measured quantities such as water quantity, with measurable physical parameters that are considered important in a particulate process. Most of these statistical models are stochastic and applicable for single storm event. These models may include climatic characteristics such as rainfall intensity and catchment parameters (imperious area, land-use type, catchment slope etc). One example of linear regression model given by Neter et al., 1990 is:

\[
Y = \beta_0 \prod_{i=1}^{n} X_i \beta_i
\]

In which \(Y\) is dependent variable, \(X_i\) are explanatory or observer variables and \(\beta_i\) are the known regression coefficients, is a common statistical model used for modelling both water quality and quantity. A unit hydrograph analysis is typically applied to a single event. However the time series may include several discrete or complex wet weather events. Equation 1 may be expanded and expressed in matrix form:

\[
\begin{bmatrix}
X_1 \\
X_2 \\
X_3 \\
\vdots \\
0
\end{bmatrix}
\begin{bmatrix}
\beta_0 \\
\beta_1 \\
\beta_2 \\
\vdots \\
\beta_M
\end{bmatrix}
\begin{bmatrix}
y_1 \\
y_2 \\
y_3 \\
\vdots \\
y_N
\end{bmatrix}
\]

Similarly nonlinear regression model is

\[
Y = \beta_0 \prod_{i=1}^{n} X_i \beta_i
\]

Where \(X\) is the matrix of measured precipitation, in which \(X_i\) is recorded at the end of the first time step, \(X_i\) is recorded at the end of second time step, etc. \(U\) is a vector of unit hydrograph ordinates and \(Y\) is a vector of estimated runoff values.

Apparent drawback of these types of models is they are developed from a given data set collected from a particular region that reflects particular spatial arrangement. For any markedly different spatial pattern and processes new data and new statistical relationship must be developed. Furthermore, the model can not consider dry weather period take into account. Because of the limitation the statistical models have been applied mainly for the crude analysis or in the situation where deterministic approaches can not be used because of insufficient data or resources. Examples of statistical models used in
urban watershed modelling can be found in Tasker and Driver,23) Jewell and Adrian,30) Driver and Troutman,31) Yao and Terakawa,32) Khan and See.33)

Linder and Ellis34) applied multiple linear-regression equation for stormwater runoff volume and peak flows in Denver, Colorado. Driver and Tasker31) applied the regression models in arid western states in USA. They emphasized regression based model in which models are calibrated to flood- frequency determination at gaged locations is the most accurate and reproducible. They developed regression models that related storm-runoff loads and volumes to easily measured physical, land-use and climatic characteristics.

Yao and Terakawa32) applied regression model to a large Fuji river basin in Japan for meteorological parameters estimation. They applied 16 years monthly data was applied to determine the parameters and found the model good agreement with the observed data which they applied in distributed model for the runoff estimation.

4.2. Dynamic Wave Equation

The flood flow is unsteady- as the flow properties (depth and velocity) changes with time, is gradually varied, because such change with time is gradual. The basic flow-governing equations are the dynamic wave equations, often referred to as the St. Venant equations or shallow water equations. These consist of the equations of continuity and momentum for gradually varied unsteady flow, respectively, expressed as:

\[ \frac{\partial h}{\partial t} + \frac{\partial Q}{\partial x} = 0 \] (continuity) (4)

\[ \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + g \frac{\partial h}{\partial x} = g(S_o - S_f) \] (momentum) (5)

Local Convective Pressure Gravity Friction
acceleration acceleration force force force
term term term term term

where, \( h \) is flow depth (m), \( Q \) is the flow per unit width (m³/sm), \( u \) is water velocity (m/s), \( g \) is acceleration due to gravity (m/s²), \( S_o \) is the bed slope (m/m), \( S_f \) is the energy gradient (m/m), \( t \) is the time (s), \( x \) is the longitudinal distance (m).

Major problem for the dynamic wave equation is that there is no analytical solution for the above equations. Approximate numerical solutions of these two equations have been used in many models such as US Army Corps of Engineers, Unsteady flow through a full NETwork of open channels (UNET) model35) and National Weather Service’s OPERational Dynamic Wave (DW-OPER) model, FLO-2D model. Aronica and Lanza36) applied dynamic wave equation in their modelling work and emphasized the importance of micro-topography for better performance. Liu and Shao37) also worked on St. Venant equation to predict the runoff in urban district in China.

4.3. Diffusive Wave Equation

The diffusive wave equation consists of the continuity and simplified momentum equation, the equation was earlier applied by Cunge et al., 1980 which was further elaborated by Singh.38)

\[ \frac{\partial h}{\partial t} + \frac{\partial Q}{\partial x} = q \] (continuity equation) (6)

\[ \frac{\partial h}{\partial x} = S_o - S_f \] (7)

where, \( q \) is the lateral flow per unit width and per unit length (m³ s⁻¹ m⁻¹ m⁻¹)

In diffusive wave equation, the term \( \frac{\partial h}{\partial x} = g(S_o - S_f) = 0 \)
resulting in \( \frac{\partial h}{\partial x} = (S_o - S_f) \).

The continuity equation includes lateral inflow. The equation is capable of simulating the attenuation in the flow because pressure slope is included in the momentum equation. A disadvantage of this equation is that there is no analytical solution of the above mentioned diffusive equations. Watershed models CAS-C2D and MIKE SHE use approximate numerical methods to solve the above equations for overland flow and channel flow. Manning equation is used to compute flow which is expressed as:

\[ Q = \frac{1}{n} AR^{1/2} S_f^{1/2} \] (8)

where, \( n \) is Manning’s roughness coefficient, \( A \) is flow cross-sectional area per unit width (m²/m), \( R \) is hydraulic radius.

4.4. Kinematic Wave Equation

As we stated above the dynamic wave equation has numerical solutions. Depending on the accuracy desired, alternative flood routing equations are generated by using the complete continuity equation while eliminating some terms of the momentum equation. These dynamic wave equations have not been used in watershed models because of their computationally intensive equations on a limited basis.

The first application of kinematic wave theory to pollutant transport was made by Brazil et al.39) They applied the theory to simulate non-point pollutant in overland flow. Later on Snyder and Woolhiser40) elaborated the equation by including the infiltration effect. Akao41) developed a kinematic wave model for pollutant wash-off by the overland flow on impervious surfaces. Havis et al.42) partitioned solute transport between infiltration and overland flow under rainfall. Singh et al.43) proposed one dimensional kinematic wave equation for the pollutant transport where he primarily assumed the pollutant transport by
Stormwater is fully mixed in the runoff water either instantaneously or in a finite period of time. The equations were further elaborated and several solutions were made by Singh, Deng et al., and Guo.

In Kinematic wave, the friction force term $\frac{\partial h}{\partial t} + \alpha n h^{n-1} \frac{\partial h}{\partial x} = q$ (9) with

$$Q = uh = c \alpha h^n, \quad u = c \alpha h^{n-1} = c \alpha h^{n-1} = nu$$

where $h$ is the depth of the flow (L), $C$ is wave celerity, $q$ is the constant rate of rainfall excess, $Q$ is discharge per unit width ($L^2/T$), $q$ is rainfall intensity (L/T), $t$ is time (T), $x$ is space coordinate (L) positive in the direction of flow, $n$ is exponent, and $\alpha$ is a depth-discharge coefficient or kinematic wave resistant parameter. For example if Darcy-Weisbach formula is used for laminar flow then

$$\alpha = 8gS_o \frac{C}{v}$$

Where $C$ is the laminar flow resistance factor and $v$ is kinematic viscosity of water and $m=3$.

If the Manning equation is employed then

$$\alpha = \left[ \frac{k}{n} \right]^{1/2} S_o^{1/2} \text{ and } m=5/3.$$

A very important kinematic flow parameter is the time at which the overland flow reaches the equilibrium under constant rate of rainfall excess. The time to equilibrium can be determined using;

$$t_e = \frac{L^{1/m}}{(\alpha_o^{-m-1})^{1/m}}$$

Where,

$t_e$ is time to equilibrium, $L$ is the length of overland plane, $i_o$ is constant rate of rainfall excess

Many watershed models such as DWSM, KINEROS, and PRMS are based on the kinematic wave equations. KINEROS and PRMS use approximate numerical solutions while DWSM uses analytical and an approximate shock-fitting (closed form) solution.

4.5 Hydrological Linear Reservoir Model

Hydrological model ignores the spatial variability in the problem. They are generally based on the conservation of mass only. The unit hydrograph, lumped continuity or storage models, the Muskingum method and nonlinear storage are considered here to be hydrological methods. Some hydrological models can be interpreted as hydraulic models. The Muskingum method is one approach that can be described as an approximation to the shallow water wave equations or in terms of the conservation of mass.

There are two types of model: linear and nonlinear. In linear instantaneous (unit hydrograph) it is assumed that the catchment acts as a reservoir and the outflow is a linear function of storage;

$$S = KO$$ (10)

In which $S$ is the storage, $O$ is the outflow and $K > 1$ is a constant storage coefficient. Combined with the continuity equation for the reservoir.

$$\frac{ds}{dt} = I - O$$ (11)

where, $I$ is the inflow, the exponential form of the instantaneous unit hydrograph for a single storage is

$$O(t) = \frac{1}{K} \exp(-t / K)$$ (12)

A large catchment can be subdivided into equal subcatchments with each subcatchment considered as a separate linear storage. The instantaneous unit hydrograph for a cascade of non-linear reservoir is given by

$$O(t) = \frac{1}{K(n-1)!(Kt)^{n-1}} \exp(-t / K)$$ (13)

which resembles a Gamma function. This model is linear because $K$ is constant and does not consider translation of the flow.

In case of continuous or storage model, it satisfies the conservation of mass. The catchment response is instantaneous because the momentum equation is completely ignored. Replacing the spatial derivatives in Equation $\partial Q / \partial t = \partial Q / \partial x = q$ with finite differences so that $\partial Q / \partial x = (I - O) / \Delta x$ then

$$\frac{ds}{dt} = I - O$$

In which is the storage $S = A\Delta x$. This equation is known as the storage equation which is used in simple routing methods.

4.6 Storage-based or Non-linear Reservoir Equations (continuous)

Nonlinear models, the storage is expressed as a nonlinear function of outflow so that

$$S = KO_w^m$$ (14)
where \( m \) is some power. Substituting into the discretised storage,

\[
O_w = XI + (1 - X)O
\]  

(15)

and models which include translation\(^{49}\) have been developed.

Many of the models such as ANSWERS, ANSWERS-Continuous, HSPF, INFOWORKS use the simple storage-based equations for flow routing. The equation consists of the spatially uniform and temporarily variable continuity equation and a flow equation expressed in terms of channel (or plane) roughness and geometry, such as Manning’s equation, as expressed below:

\[
\frac{ds}{dt} = I - O
\]  

(16)

\[
Q = \frac{1}{n} AR^{2/3} So^{1/2}
\]  

(17)

where,

\( s \) is the storage volume of water (m\(^3\)), \( I \) is the inflow rate (m\(^3\)/s) and \( O \) is the outflow rate (m\(^3\)/s).

4.7. Curve Number and Empirical Equation

Soil Conservation Service developed Curve Number (CN) as an index combining hydrologic soil group and land use factors (cover and condition). SCS curve numbers are used to estimate the amount of precipitation which becomes runoff, and the amount which infiltrates into the soil.

The curve numbers are selected from tabulated values for fallow or appropriate land use, treatment, and hydrologic conditions (crop condition) plus an antecedent moisture adjustment. Runoff and infiltration volumes can be calibrated by entering override curve numbers for a field. The standard SCS-CN method\(^{50}\) The SCS Curve Number model is mainly applied for the agricultural condition.

Many of the models, such as SWAT, AGNPS and AnnAGNPS, do not route water using mass conservation based continuity equations as described above. SWAT and AnnAGNPS maintain water balance through accounting daily or subdaily water budgets. All three of them use the USDA Soil Conservation Service runoff curve number method\(^{51}\) to compute runoff volumes and other empirical relations similar to the Rational formula\(^{52}\) to estimate peak flows, which may be expressed as:

\[
Q_r = \frac{(P - 0.2S_r)^2}{P + 0.8S_r}
\]  

(18)

\[
S_r = \frac{254000}{CN} - 254
\]  

(19)

\[
Q_F = 0.0028CI\theta
\]  

(20)

where, \( Q_r \) is direct runoff (mm), \( P \) is accumulated rainfall (mm), \( S_r \) is potential difference between rainfall and direct runoff (mm), \( CN \) is curve number representing runoff potential for a soil cover complex (values 2 to 100), \( Q_p \) is peak runoff rate (m\(^3\)/s), \( C \) is the runoff coefficient (values 0.02 to 0.95), \( i \) is rainfall intensity (mm/h), and \( A \) is watershed area (ha).

Jacobs et al.\(^{53}\) used SCS Curve Number to improve rainfall/ runoff estimation in Oklahoma by including the remotely-sensed soil moisture. Ikenberry et al.\(^{54}\) worked on CN number and concluded that good elevation data set is important for effective measurement of runoff depth, runoff volume and peak discharge. Barros et al.\(^{55}\) applied the SCS Curve Number method to predict the peak flood runoff for Sao Paolo.

4.8. Analytical Probabilistic Models

This model provides an alternative approach to the analysis of urban drainage. The primary benefit of the model lies in the availability of explicit mathematical solutions to performance measures. A range of system design element or required performance levels can be investigated with ease while incorporating the full range of meteorological conditions. Early work in the model was done by Howard\(^{56}\) then elaborated by Adams and Bontje\(^{57}\) the analytical models using derived probability distribution theory. Chen and Adams\(^{24}\) recently explained more about the model. To develop the expression for runoff quantity control, the input precipitation (\( v \)) is transformed into runoff (\( v_r \)) using the following relationship:

\[
v_r = \begin{cases} 
0 & ; v \leq S_d \\
\Phi(v - S_d) & ; v > S_d 
\end{cases}
\]  

(21)

where, \( v_r \) is the runoff volume (mm), \( v \) is the rainfall volume, \( S_d \) is the depression storage on the catchment expressed as an equivalent uniform depth across the entire catchment (mm), and \( \Phi \) is the runoff coefficient.

A catchment reacts to rainfall by first filling the depression storage volume, \( S_d \), prior to runoff generation. The rainfall volume exceeding depression storage (\( v - S_d \)), is multiplied by the runoff coefficient, \( \Phi \), to determine the runoff volume \( v_r \). The total losses, \( S_d \) and \((1 - \Phi)(v - S_d)\) can be viewed as a combination of infiltration and some evaporation. In analytical model, the number of average annual runoff events, \( n_R/yr \), outlines the number of rainfall events which actually generate surface runoff to be routed through a stormwater drainage system and is described by

\[
n_R = \theta e^{-S_d} 
\]

Where, \( \theta \) is the average annual number of rainfall events and \( \zeta \) is the inverse of the mean rainfall event volume (1/mm).
The average annual surface runoff volume washing off the catchment, \( R \) (mm/yr) is described by:

\[
R = \frac{\Phi}{\zeta} e^{-\zeta S_o}
\]

And the probability per event of a spill of any magnitude is given by

\[
G_p(0) = \left[ \frac{\lambda/\Omega}{\lambda + \zeta/\Phi} \right] \left( \frac{\psi/\Omega + \zeta/\Phi e^{-\psi/\Omega + \zeta/\Phi}}{\psi/\Omega + \zeta/\Phi} \right) e^{-\zeta S_o}
\]

(22)

where, \( \lambda \) is the inverse of the mean rainfall event duration (1/hr), \( \psi \) is inverse of the mean inter-event time (1/hr), \( \Omega \) is the controlled release rate from storage (mm/hr), \( S_o \) is the active storage volume averaged over catchment area (mm) and \( G_p(0) \) is probability per rainfall event of any spill of any magnitude.

5. Conclusion

This paper reviews mathematical methods used in stormwater modelling and has been undertaken by examining a number of models that are in current use. Generally, all the models are simplified as conceptual or empirical depending on whether the model is based on physical laws or not. In both cases if any of the variables in the model are regarded as random variables having a probability distribution, then the model is a stochastic model. Otherwise the model is deterministic (based on process descriptions). The analytical techniques are presented in this paper.

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